Greek Letters

$\dot{\gamma}$ = shear rate
η = shear viscosity
η_o = reference viscosity
η_T = temperature function defined in Equation (29)
ρ = density
τ = shear stress
Φ = derived function as defined in Equation (7)
χ = x coordinate of a isopressure line
Ψ = stream function
Ψ_o = constant, see Equations (16a) and (16b)
T_w = mold wall temperature
T_c = melt temperature (or barrel temperature)

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Turbulent Drag Reduction in Dilute Fiber Suspensions: Mechanistic Considerations

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Both fibrous and polymeric additives, used alone or in combination, appear to affect primarily the fluid in the sublayer region adjacent to a solid surface, contrary to previous predictions. An analysis of fluid deformation modes shows tentatively why drag reduction levels become less sensitive to system scale when fibrous additives are employed, and why polymeric and fibrous additives may be more effective in combination than when either additive is used alone.

SCOPE

Drag reduction, under turbulent flow conditions, has been observed in solutions of synthetic as well as natural polymers, in suspensions of solid particles, and in micellar systems. Comprehensive and readable accounts have been provided in recent years by Savins (1967, 1969), Kenis (1968), Hoyt (1972*a*, 1972*b*), Little et al. (1975), and Virk (1975). Mechanistic interpretations of the phenomenon in polymer solutions have centered on the viscoelastic (nonlinear) fluid properties for many years (Dodge and Metzner, 1959; Metzner and Park, 1964; Astarita, 1965; Gadd, 1965). Seyer and Metzner (1966, 1969) introduced arguments concerning the high resistance of viscoelastic media to elongational deformations, and Hansen (1973) and Ruckenstein (1971) focused upon the low resistance to deformation of viscoelastic media when deformed on time scales which are small in comparison to the natural time of the medium.

Suspensions of solid particles, especially fibrous ones, exhibit a number of pragmatic advantages over polymers when use of additives is considered for drag reduction purposes: they offer a much greater resistance to degradation, their efficacy is less sensitive to the scale of the process, and they may be removed from the fluid more

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easily. Higher concentrations may be required for a given level of drag reduction in small tubes, but this difference decreases with increasing scale of the system, and it may also be partly offset by differences in the unit cost of the additives. Blatch (1906) and Forrest and Grierson (1931) *authored early* papers which dealt with drag reduction in suspensions; recent studies by Bobkowicz and Gauvin (1965), Kerekes and Douglas (1972), Hoyt (1972*b*), Vaseleski and Metzner (1974), and Radin et al. (1975) serve to describe the current state-of-the-art.

The mechanistic origins of drag reduction in fiber suspensions do not appear to be as clear as in the case of polymer solutions; neither is the origin of the dramatic synergism between fibrous and polymeric additives reported by Lee et al. (1974) and Kale and Metzner (1974). The purpose of the present work is to contribute to a mechanistic understanding of these problems.

CONCLUSIONS AND SIGNIFICANCE

A consistent interpretation of the parameters in the drag reduction equations is obtained from independent measurements of the velocity profile in the turbulent core and of the drag coefficients in both smooth and rough tubes. These show that the principal drag reducing mechanisms in dilute fiber suspensions are centered in the wall region, just as in the case of polymeric systems. The reasons for an earlier contrary report are noted; this result is important in the utilization of drag reduction in external flows as encountered in naval applications, since smaller amounts of additives are required for modification of the fluid in the wall region than for treatment of the entire turbulent boundary layer.

Plausible but unproven suggestions are advanced in explanation of two important phenomena: the independence of drag reduction level upon system scale in fiber suspensions and the synergism which may occur when both fibrous and polymeric additives are employed together.

THEORETICAL CONSIDERATIONS

It is well known (see, for example, the treatise of Schlichting, 1968) that the velocity profile for Newtonian fluids under turbulent flow conditions and in hydraulically smooth pipes may be approximated by

$$u^+ = 2.5 \ln y^+ + 5.5 \tag{1}$$

over most of the turbulent core. This equation may be written in general as

$$u^+ = A \ln y^+ + B \tag{2}$$

Very close to the wall the flow conditions are better described as being those of disturbed laminar flow, and the velocity profile is closely approximated by

$$u^+ = y^+ \tag{3}$$

It is convenient to consider turbulent pipe flow to be comprised of only two regions, a wall region and a turbulent core, and to use Equations (3) and (2), respectively, to describe the velocity profiles. Integrating these profiles to obtain the volumetric flow rate and converting this into a friction factor-Reynolds number relationship, one obtains

$$\frac{1}{\sqrt{f}} = \frac{A(1-\xi_l)^2}{\sqrt{2}} \ln (Re\sqrt{f}) + \frac{[B - A \ln 2\sqrt{2}](1-\xi_l)^2 - G}{\sqrt{2}} \tag{4}$$

At high Reynolds numbers ($> 10^4$), the viscous sublayer may be negligibly small in thickness in comparison to the pipe radius for Newtonian fluids and is usually neglected. The above equation may then be written as

$$\frac{1}{\sqrt{f}} = \frac{A}{\sqrt{2}} \ln (Re\sqrt{f}) + \frac{B - A \ln 2\sqrt{2} - G}{\sqrt{2}} \tag{5}$$

The measurement of u^+ as a function of y^+ and of the friction factor over a range of Reynolds numbers should give consistent values for the parameters A and B in the above equations. For polymeric solutions, the parameter B has been found to be a function of the stress level or friction velocity (Elata, 1966; Meyer, 1966; Rudd, 1969; Seyer and Metzner, 1969), while the parameter A remains unchanged from its value for Newtonian fluids.

In the case of flow through rough pipes, the equivalent of the parameter B of Equation (2) becomes a function of the roughness of the pipe wall. For fully rough flow conditions, in which the friction factor is independent of Reynolds number, one may write

$$u^+ = A \ln y^+ + D_1 \tag{6}$$

in which

$$D_1 = B_1 - A \ln \frac{k_s u^*}{\nu} \tag{7}$$

Equations (6) and (7) may be combined to give

$$u^+ = A \ln \frac{y}{k_s} + B_1 \tag{8}$$

The value of the parameter A in Equations (2) to (8) is found to be identical in all cases. In the fully rough region, $B_1 = 8.5$ for Newtonian fluids. Integrating the above velocity profile equation and converting it into the friction factor-Reynolds number relationship, one obtains

$$\frac{1}{\sqrt{f}} = 1.628\, A \log (R/k_s) - 1.061\, A + \frac{B_1}{\sqrt{2}} \tag{9}$$

Thus it is possible to relate smooth and rough tube information for both velocity profiles and friction factors and perform consistency tests.

EXPERIMENTAL

Velocity profiles were measured in a 5.0 cm transparent acrylic pipe, and pressure drop-flow rate results were obtained in 2.4 and 5.0 cm (I.D.) pipes. The general characteristics of the flow loop have been described previously by Vaseleski and Metzner (1974).

Since the fluid opacity prohibited use of optical methods for determining point values of the time-average velocities, a pitot tube method was employed, with a device as identical as possible to that used successfully by Mih and Parker (1967). In this design the pitot tube used as a pressure sensor is kept from being plugged by the fibrous suspension by means of a purge-water stream through the annulus formed between the pressure sensing tube and a concentric larger one. If the flow rate of the purge-water stream is too low, the reading obtained

TABLE 1. COMPARISON BETWEEN MEASURED AND INTEGRATED MEAN VELOCITIES

	Actual velocity cm/s	Integrated mean velocity (from pitot tube readings) cm/s
Water	260.0	259.37
	184.2	185.53
	134.5	137.98
Asbestos 1 000 p.p.m.	245.5	247.33
	189.5	191.12
	132.8	134.43
Asbestos 2 500 p.p.m.	143.0	144.34
	266.0	269.50
Mixed systems 1 000 p.p.m. asbestos + 150 p.p.m. polymer	149.1	150.08
	263.1	268.20
2 500 p.p.m. asbestos + 100 p.p.m. polymer	142.5	145.76

is low, and thus plugging of the pitot tube may result. On the other hand, if the purge-water flow rate is excessively large, it causes suction at the tip of the probe and also leads to a spurious (negative) reading. Between these two limits, however, a stable and constant reading of the impact pressure is obtained. For each position of the pitot tube, at least two different purge velocities were used, and constancy of the observed pressure reading ensured that the proper range of flow rates was being employed. The total mass of fiber suspension used on the flow loop was so large (about 725 kg) that addition of this purge stream did not change the concentration of the system appreciably.

Alternate pitot tube techniques have been reported by Brecht and Heller (1950) and by Daily and Bugliarello (1961), but these were not investigated in any detail as both tend to increase the size of the measuring element and, under some conditions, appear to lead to spurious results even for water. In the present work, the variation of velocity over the diameter of the measuring element, arising from the finite slope of the velocity profile, was always less than 1%.

Fiber concentrations studied were 1 000 and 2 500 p.p.m. (by weight) of Canadian J-M asbestos 3T12. As a dispersant, Aerosol OT, 0.25% by weight, was used in all systems. Weighed amounts of fibers were added to a concentrated solution (approximately 1% by weight) of Aerosol OT to prepare a small volume of concentrated solution. This in turn was slowly added to the main storage tank containing weighed amounts of water with dissolved dispersant which was kept circulating through the system with a Moyno pump. Gentle stirring with a marine propeller mixer was also employed to maintain the homogeneity of the suspensions. The contents of the storage tank were then circulated for about 9 hr at medium flow rates and for about 3 hr at the highest possible flow rate. This degraded the test fluid sufficiently to stabilize it over the period of experimental measurements.

Mixed systems contained dissolved polymer as well as suspended fibers. The required amount of polymer was dissolved in water to make a fresh, concentrated, and homogeneous solution. This, then, was slowly added to the suspension and throughly mixed. Again, the systems were degraded by rapid mixing to achieve a stabilized test fluid.

For the experiments in rough pipes, two 1 in. (nominal) brass pipes were knurled from the inside. The inner surfaces developed in this manner consisted of a 20-pitch medium diamond pattern with a depth of 0.81 mm in one case and a 33-pitch fine diamond pattern having a depth of 0.51 mm in the other. (The pitch of a knurling tool means the number of teeth per linear inch.) Each roughness element is a rectangular parallelpiped in elevation; the parallelepiped has a diamond shaped cross section. The direction of the flow is oblique to the sides and corners of the roughness element, the obliqueness angle depending on the pitch of the threads cut by the

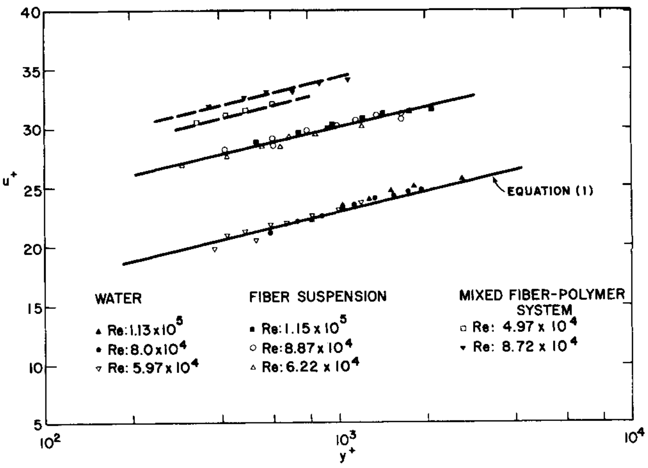

Fig. 1. Velocity profiles for water, for a 1 000 p.p.m. asbestos suspension and for a mixed system containing 1 000 p.p.m. asbestos plus 150 p.p.m. polyacrylamide in solution.

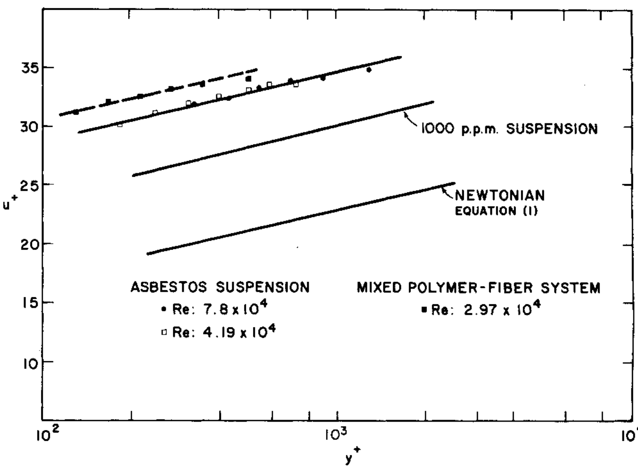

Fig. 2. Velocity profiles for a 2 500 p.p.m. asbestos suspension and a mixed system containing 2 500 p.p.m. asbestos plus 100 p.p.m. polyacrylamide. Newtonian and 1 000 p.p.m. lines from Figure 1 shown for comparative purposes.

knurling tools. The dimensionless roughness R/ks, as characterized by friction factor measurements with water at high Reynolds numbers, was 27.5 for one tube and 53.5 for the other [Equation (9)]. In this part of the work, the experimental test fluids contained 500 p.p.m. of the asbestos fibers and 0.25% dispersant. The polymer-fiber mixture studied contained 25 p.p.m. of polymer in addition to 500 p.p.m. of fibers.

RESULTS AND DISCUSSION

The velocity profile data are shown in Figures 1 and 2. As seen in Figure 1, the results obtained for water check very well with Equation 1. In each case the velocity profile obtained was also integrated and checked against the actual measured flow rate. This comparison between the integrated velocity profile and the actual mean velocity is given in Table 1. Collectively, the water data provide confidence in both the precision and reproducibility of the measurements.

The data shown in Figures 1 and 2 reveal that all profiles exhibit the same slope in the turbulent core; that is, the velocity profile parameter A of Equation (2) is the same for fiber suspensions and for fiber-polymer systems as for the Newtonian carrier fluid. This contrasts to the results reported by both Daily and Bugliarello (1961) and Mih and Parker (1967) who found that the profile becomes flatter as the center line is approached (or as y^+ increases). As their fiber suspensions were generally suf-

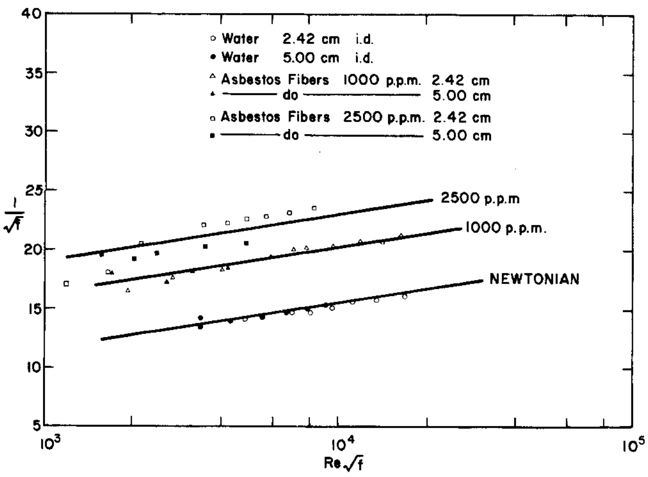

Fig. 3. Flow rate-pressure drop data for asbestos fiber suspensions compared to water data with 2.42 and 5.00 cm. (I.D.) tubes used.

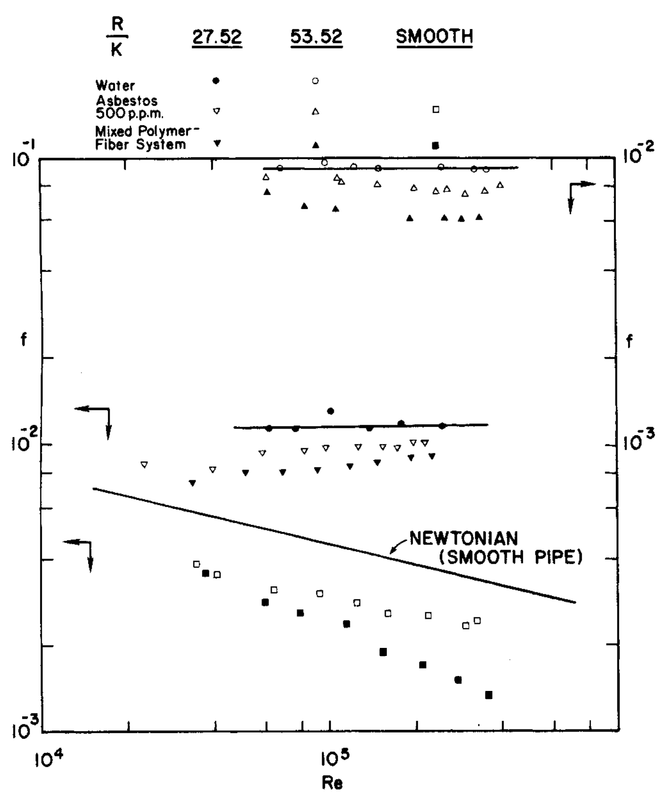

Fig. 4. Friction factor-Reynolds number results for rough pipes. Asbestos suspension contains 500 p.p.m. of fibers; the mixed system contains 500 p.p.m. of fibers and 25 p.p.m. of polymer.

ficiently more concentrated to exhibit yield values larger (in comparison to the shear stress at the wall) than those in our work, this difference is not surprising. Such more concentrated suspensions, with flatter profiles in the turbulent core, and therefore steeper wall-region profiles, tend not to be as drag reducing, however, and so their behavior is of less interest in the present context.

For both the 1 000 and 2 500 p.p.m. suspensions, the data at various Reynolds numbers superimpose, showing that the parameter B is independent of the stress level to which the fluid is exposed, at least over the range of Reynolds numbers or stress levels studied. By contrast, in the turbulent flow of polymer solutions this parameter is known to be dependent on stress levels or, equivalently, upon both Reynolds numbers and tube diameter. This is discussed, for example, by Hoyt (1972*a*). Figure 1 shows that such dependence is carried over into the behavior of mixed polymer-fiber systems. The fact that the data for the mixed systems fall above those for the suspension alone means that both the actual drag reduction and the numerical value of the parameter B are larger in mixed systems than in suspensions.

Table 2. Comparison of the Values of A and B Obtained from Velocity Profile and Friction Factor Data

	Velocity profile		Friction factor data	
	A	B	A	B
Water	2.5	5.5	2.5	5.5
1 000 p.p.m. asbestos	2.5	12.5	2.5	12.25
2 500 p.p.m. asbestos	2.5	17.25	2.5	16.7 (mean line) 17.5 (data at high Reynolds numbers)

Since the parameters A and B appear to be constants for a given suspension, independent of Reynolds number or stress level, one should be able to discern these same parameters, with precision, from independent pressure drop-flow rate measurements plotted in the format suggested by Equation (5). Figure 3 shows such a plot, and Table 2 compares the least-squares determination of the numerical values of the parameters. It is seen that there is a very high level of consistency between these independent measurements. Figure 3 also shows an additional diameter or stress level effect. This may arise from the assumption that the parameter G of Equation (5) may be taken as a constant when, in fact, a small dependency on stress level or tube diameter may be expected, as noted by Seyer and Metzner (1969). This dependency tends to obscure the true values of A and B when they are obtained from pressure drop-flow rate results as in Figure 3 unless data for various tube diameters overlap generously. It is believed that this difficulty led Vaseleski and Metzner (1974) to conclude, erroneously, that the parameter A increases as the concentration level of the suspension is increased.*

Fiber suspensions which are drag reducing under conditions of turbulent flow in smooth pipes are also drag reducing in rough pipes (Figure 4), and the Reynolds numbers required to reach the asymptotic conditions of constant friction factors may be increased somewhat. The mixed system yields greater levels of drag reduction than does the fiber alone; that is, the synergism between the polymeric and fibrous additives occurs in rough pipes as well as in smooth ones. Since the present velocity profile studies have shown that the value of the parameter A for suspensions is the same as that for Newtonian fluids, Equation (9) may be used to yield information on the value of B_1. By using the present results, with $A = 2.5$, the values of B_1 obtained from the two rough pipes, for the fiber suspension, are 9.75 and 9.79. The two values check very well with each other. The Newtonian value of B_1 is 8.5; the drag reduction levels obtained in rough pipes, with a given fluid, are not as great as in smooth tubes.

MECHANISTIC CONSIDERATIONS

Following prior arguments based upon the role of the Townsend-Bakewell roll waves in the wall regions of turbulent shear flows (Seyer and Metzner, 1969), the present results suggest that fibrous as well as polymeric additives may interfere with this roll-wave structure. The

* This may occur, and probably does, in more concentrated suspensions. The present work shows that fibrous additives may exert a separate influence in the wall region, however.

resistance to the extension of polymer solutions is given by Denn and Marrucci (1971) for a contravariant Maxwell model:

$$\mu_e = 3\mu \frac{2}{3(1-2\,\theta\Gamma_e)}\left[1 - e^{-(1-2\theta\Gamma_e)t/\theta}\right] + \frac{\left[1 - e^{-(1+\theta\Gamma_e)t/\theta}\right]}{3(1+\theta\Gamma_e)} \quad (10)$$

Under conditions of rapid deformation rate, as of primary interest in turbulent fields, the term $(1 - 2\,\theta\Gamma_e)$ will be negative, and the resistance to extension, at a given deformation rate, is predicted to rise exponentially in time and may reach very high levels. Such high levels have been observed by a number of investigators in studies of the dilute solutions of interest in drag reduction (Astarita and Nicodemo, 1970; Metzner and Metzner, 1970; Baid, 1973). Although Equation (10) does not portray those results precisely, the general form appears to be approximately correct. Thus, two dimensionless groups, t/θ and $\theta\Gamma_e$, determine this resistance to deformation. Following Seyer's analysis of the Townsend-Bakewell roll-wave kinematics, Γ_e is proportional to the friction velocity, and polymeric additive may be less effective in large scale systems, at a given Reynolds number, simply because the friction velocity, hence the term $\theta\Gamma_e$, decreases. The role of the group t/θ is not clear at present.

The behavior of fiber suspensions in elongational flows is quite different (Batchelor, 1971; Kizior and Seyer, 1974; Mewis and Metzner, 1974). For the case of long slender fibers suspended in a Newtonian liquid, these authors show that

$$\mu_e = 3\mu\left[1 + \frac{4}{9}\,\frac{c\,\alpha^2}{\ln(\pi/c)}\right] \quad (11)$$

In this case there is no dependence of μ_e on the deformation rate, and, if Equation (11) applies under the transient conditions of interest, the roll waves should be equally inhibited at all stress levels, as is found to be the case.

For a given suspension, the term $c/\ln(\pi/c)$ gives the concentration dependence of the extra resistance to elongational deformations. Figure 5 plots both this term and the increased drag reduction, as measured by the parameter B of Equations (1) to (5), vs. concentration. The parameter B is shown for the data taken in the present study as well as recomputed from the Vaseleski-Metzner results for $A = 2.5$. The resemblance of the two curves is remarkable. Clearly, this qualitative resemblance of two functions cannot be used to verify the assumption of a mechanistic connection between the two phenomena. It is, speculatively, certainly indicative of the same, however.

If one accepts this hypothesis, one is also led rather naturally to a possible explanation of the synergism between polymeric and fibrous additives, as follows. The presence of fibers in a polymeric solution causes an element of the fluid suspension which is being stretched to be sheared simultaneously as a result of the flow of the solution past the rigid fibers. It is difficult to predict precisely the effects of both shearing and extensional flows upon the resistance of the fluid to deformation in a turbulent field, but a number of limiting special cases may be considered readily. The easiest calculation, and perhaps also the most pertinent result, is obtained by considering either a sheet or a cylindrical mass of fluid initially at rest which, for times greater than zero, is subjected to a velocity field such that the fluid is deformed at constant shearing and extensional deformation rates equal Γ and Γ_e, respectively. Lornston (1975) has computed the ensuing stress levels for a fluid described mathematically by the same contravariant Maxwell model used to obtain Equa-

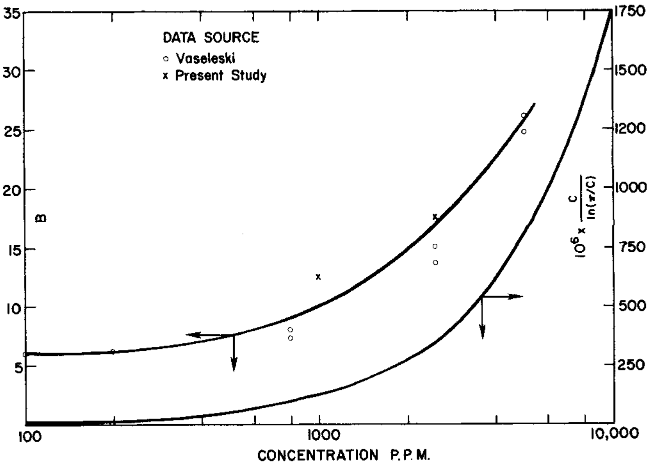

Fig. 5. Comparison of increases in drag reduction with fiber concentration, as measured by the parameter B, with increases in the resistance to elongational deformation, as determined by the term $c/\ln(\pi/c)$ All data are for JM asbestos 3T12 suspensions.

tion (10). The result, for large values of $\theta\Gamma_e$, appears to be to increase the resistance to elongational deformation by a term of order $[1 + \theta\Gamma_e(\Gamma/\Gamma_e)^2]$. Since Γ/Γ_e may be expected to be much greater than unity, at least under some conditions, the additive term could be a very large one. This would reduce radial momentum transport rates by reducing the intensity of the Townsend-Bakewell roll waves, thereby explaining the nonlinear synergism of some of the additive systems studied earlier.

ACKNOWLEDGMENT

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NOTATION

A = slope of logarithmic velocity profile in Equation (2)
B = intercept parameter defined in Equation (2)
B_1 = intercept parameter defined in Equation (8)
c = concentration (volume fraction) defined in Equation (11)
d = diameter of fiber
D = diameter of pipe
D_1 = parameter defined in Equation (6)
f = friction factor $\tau_0/\frac{1}{2}\,\rho v^2$
G = constant of integration in Equation (4)
k_s = height of equivalent roughness element
l = length of fiber
R = radius of pipe
Re = Reynolds number $DV\rho/\mu$
S_{ij} = components of the total stress tensor
u = time-averaged mean axial local velocity
u^* = friction velocity $\sqrt{\tau_0/\rho}$
u^+ = u/u^*
V = mean velocity through pipe
y = distance away from the wall in the radial direction
y^+ = yu^*/ν

Greek Letters

α = aspect ratio of fiber (l/d)
Γ = shear rate
Γ_e = stretch rate
θ = relaxation time of the fluid
μ = shear viscosity of fluid
μ_e = resistance to extension, or extensional viscosity, taken as $\mu_e \equiv \dfrac{S_{11} - 1/2(S_{22} + S_{33})}{\Gamma_e}$

ν = kinematic viscosity μ/ρ
ξ_l = dimensionless distance y/R at which Equations (2) and (3) coincide
ρ = density of fluid
τ_o = shear stress at the wall

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A Flow Model for Gas Movement in Spouted Beds

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A two-region model of a spouted bed, postulating vertical plug flow of gas in the spout and dispersed plug flow along curved streamlines in the annulus, is proposed. The extent of axial dispersion is accounted for by a coefficient D which is an adjustable parameter of the model. Experimental support for the theory is provided by residence time-distribution data obtained by using helium gas as tracer, covering a wide range of conditions. Values of the coefficient D, determined from a comparison between predicted and observed RTD curves, are generally higher than those reported for packed beds but much smaller than those for fluidized beds.

SCOPE

The spouted bed technique for contacting a fluid with coarse granular solids has attracted attention for a variety of applications (Mathur and Epstein, 1974), some involving heat and/or mass transfer (drying, granulation,